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DEVELOPING DOUBLE SAMPLING PLANS FOR ATTRIBUTES TO MEET  
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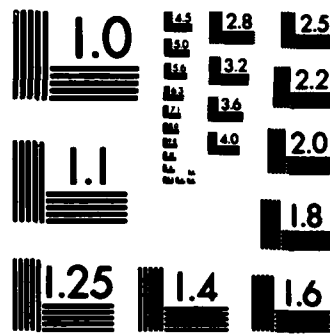
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DEVELOPING DOUBLE SAMPLING PLANS  
FOR ATTRIBUTES TO MEET SAMPLE SIZE CRITERIA

Research Report No. 84-32

by

R.W. Rangarajan  
K.B. Beitler  
and  
R.S. Leavenworth

# RESEARCH REPORT

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# ABSTRACT

This study reports on the development of a FORTRAN IV program to produce double sampling acceptance plans for attributes data. The plans must satisfy two points on an <sup>operating characteristic</sup> OC curve, the AQL point ( $p_1, 1-\alpha$ ) and the <sup>rejectable quality level</sup> RQL point ( $p_2, \beta$ ). Two models are given. MODEL I has an additional constraint that the maximum value of the ASN must be less than or equal to the sample size for a corresponding single sampling plan. MODEL II relieves this constraint. In either case, the resulting plan has a minimum ASN evaluated at the quality level  $p_1$  among all sampling plans satisfying the constraints.

## INTRODUCTION

Frequently double sampling plans are employed in lieu of single sampling plans for lot acceptance by attributes especially when lot or batch sizes are large. A number of systems of sampling plans are available the most recognized of which at least in the United States, are the MIL-STD-105D AQL systems and the Dodge-Romig LTD and AOQL systems.

In addition methods have been reported for developing tailored double sampling plans usually based on the specification of two points on an OC curve (the likelihood function). Two such procedures, largely taken from a Chemical Corps Engineering Agency (1953) publication, are contained in Tables 8-2 and 8-3 of Duncan (1974). These tables, based on the Poisson distribution, allow the tailoring of a plan to a Producer's Risk ( $\alpha$ ) of 0.05 at a designated quality level  $p_1$  and a Consumer's Risk of 0.10 at a designated level  $p_2$ . Plans may be found for which  $n_2$ , the second sample size equals  $2n_1$  and where  $n_2$  equals  $n_1$ . In either case the rejection number (cumulative) on both samples is taken to be the acceptance number on the second sample (cumulative),  $c_2$ , plus one.

In 1969 Guenther, following on some earlier work by Cameron (1952), developed what amounted to a brute-force algorithm for finding single sampling acceptance plans to satisfy two points on the likelihood function. The main difference between these two procedures is that while Cameron's procedure assured a plan with risk levels as close as possible to those designated for the quality levels  $p_1$  and  $p_2$ , Guenther's procedure assures risk levels at least as good as those stipulated. In addition, Guenther's procedure allows the use of the hypergeometric, binomial or Poisson distributions assuming adequate tables are available.

In 1970 Guenther extended his work to double sampling plans. Again the algorithm is essentially brute force employing probability tables extensively. Unlike the Chemical Corps tables, however, the fixed relationship between  $n_1$  and  $n_2$  is not required nor does the Poisson have to be used. The disadvantage of his procedure is the laborious effort required if a plan is to be developed by hand calculation. The algorithms, however, are sufficiently simple to be programmed easily for the computer. Hailey (1980) provides an ANSI Standard FORTRAN program based on Guenther's algorithm which finds the minimum sample size single sampling plan.

In this study, a program is developed for finding double sampling plans and characteristics of the average sample size functions of the plans found are compared. The basic algorithm follows that of Guenther. However, an objective function is introduced as described in the following paragraphs.

#### PROBLEM FORMULATION

When plotted as a function of  $p$ , the likelihood function  $L(p)$  of a sampling plan provides the operating characteristic (OC) curve for the plan. Using the binomial distribution as an example,  $L(p)$  for a double sampling plan is:

$$L(p) = \sum_{d_1=0}^{c_1} \binom{n_1}{d_1} p^{d_1} (1-p)^{n_1-d_1} + \sum_{d_1=c_1+1}^{r_1-1} \sum_{d_2=0}^{c_2-d_1} \binom{n_1}{d_1} \binom{n_2}{d_2} p^{d_1+d_2} (1-p)^{n_1+n_2-d_1-d_2} \quad (1)$$



where:

- $p$  = incoming proportion of nonconforming units
- $n_1$  = first sample size
- $n_2$  = second sample size
- $c_1$  = acceptance number on first sample
- $r_1$  = rejection number on first sample
- $c_2$  = acceptance number on second sample (cumulative for  $n_1+n_2$ ). The rejection number on the second sample,  $r_2$ , is  $c_2+1$ .

In this study, it is assumed that  $r_1 = r_2 = c_2 + 1$ . Thus the upper limit of the first summation,  $(r_1 - 1)$  in the double summation portion of  $L(p)$  (the probability of acceptance on the second sample), may be replaced by  $c_2$ . By so doing a double sampling acceptance plan is fully described by the plan parameters  $n_1$ ,  $n_2$ ,  $c_1$ , and  $c_2$ .

Discussion of the algorithm for finding double sampling plans will break  $L(p)$  into its two parts, the probability of acceptance on the first sample,  $Pa(n_1)$ , and the probability of acceptance on the second sample,  $Pa(n_2)$ . Thus:

$$L(p) = Pa(n_1) + Pa(n_2)$$

where for the binomial case:

$$Pa(n_1) = B(c_1 | n_1, p) = \sum_{d_1=0}^{c_1} \binom{n_1}{d_1} p^{d_1} (1-p)^{n_1-d_1}$$

$$Pa(n_2) = BB(c_1, c_2 | n_1, n_2, p)$$

$$= \sum_{d_1=c_1+1}^{c_2} \sum_{d_2=0}^{c_2-d_1} \binom{n_1}{d_1} \binom{n_2}{d_2} p^{d_1+d_2} (1-p)^{n_1+n_2-d_1-d_2}$$

If two design points are designated on the likelihood function, ideally a single  $(n_1, n_2, c_1, c_2)$  set can be found yielding an OC curve which passes exactly through these points. This would suggest setting the likelihood function for each quality level equal to its respective probabilities and solving for a set (or several sets) which exactly satisfy the equations. However, since  $n_1, n_2, c_1$  and  $c_2$  all must be integer, it is doubtful that any set can be found which gives exact equality for both equations.

In recognition of this fact the Cameron procedure for single sampling plans selects a plan which is as close as possible to the OC curve at the two points. The Guenther procedure rests on the formulation of inequality constraints that guarantee risk levels at least as good as those stipulated. It is the Guenther procedure which is used in this study.

The OC curve points selected are:

$p_1$  = An Acceptable Quality Level (AQL), following the definition in MIL-STD-105D, which should be accepted with a probability of at least  $1-\alpha$ ,  $\alpha$  being the Type I error risk.

$p_2$  = A Rejectable (poor) Quality Level (RQL) which should be accepted with no more than a low probability  $\beta$  (Type II error risk).

These two design parameters therefore may be expressed as:

$$(p_1, 1-\alpha) \text{ and } (p_2, \beta).$$

The resulting constraint equations are:

$$L(p_1) > 1-\alpha \tag{2}$$

$$L(p_2) < \beta \tag{3}$$

Actually an infinite number of double sampling plans may be found which will satisfy these two inequalities. Thus some measure of performance must be specified in order to choose among them. The measure chosen in this study was to minimize the ASN when the lots are at the AQL,  $p_1$ .

The ASN function for a double sampling plan is:

$$ASN = n_1 + n_2 P(n_2)$$

where:

$P(n_2)$  is the probability of taking the second sample.

In binomial form,

$$\begin{aligned} ASN &= n_1 + n_2 \sum_{d_1=c_1+1}^{c_2} \binom{n_1}{d_1} p^{d_1} (1-p)^{n_1-d_1} \\ &= n_1 + n_2 [B(c_2 | n_1, p) - B(c_1 | n_1, p)] \end{aligned}$$

Thus an objective function was introduced as follows:

Select  $(n_1, n_2, c_1, c_2)$  to:

$$\text{minimize } ASN(p_1) = n_1 + n_2 [B(c_2 | n_1, p_1) - B(c_1 | n_1, p_1)] \quad (4)$$

Subject to equation (2) and (3).

Two models were formulated on the basis of equations (2), (3) and (4). Model I introduced an additional constraint, along the lines of MIL-STD-105D, guaranteeing that the maximum value of the ASN of the double sampling plan,  $ASN_{MAX}$ , does not exceed the sample size  $n_s$  of the minimum  $n$  and  $c$  for a single sampling plan satisfying equations (2) and (3). This value of  $n_s$  is designated  $n_s^*$ .

In order to study the effect that the ASNMAX constraint had on the value of the objective function,  $ASN(p_1)$ , the constraint was removed in Model II and Model II was run using the same design parameters. In this way one can assess, in terms of average number of items inspected, the penalty paid by requiring that the ASNMAX not exceed  $n_s^*$ .

#### DEVELOPMENT OF THE ALGORITHM

The first step in developing a double sampling plan is to find the minimum single sampling plan  $(n_s^*, c^*)$  satisfying equation (2) and (3) where, for example:

$$L(p) = \sum_{d=0}^c \binom{n}{d} p^d (1-p)^{n-d}.$$

For any fixed value of  $c$ , there is some maximum value of  $n$  which will satisfy equation (2). If  $n$  is increased beyond this value, designated  $n_u$  for an upper bound, equation (2) will be violated. Correspondingly, there is a minimum value of  $n$ , designated  $n_l$ , which just satisfies equation (3). Any value of  $n$  less than  $n_l$  will produce a value of  $L(p_2)$  greater than  $\beta$ .

Beginning with  $c$  equals 0,  $n_l$  and  $n_u$  are found by incrementing  $n$  one unit at a time. If the resulting values of  $n_u$  is not greater than or equal to  $n_l$ ,  $c$  is increased by one unit and the procedure repeated. Eventually at some value of  $c$ ,  $n_u > n_l$  is satisfied, the current value of  $c$  becomes  $c^*$  and  $n_s^*$  is set equal to  $n_l$ . Thus the minimum single sampling plan satisfying equations (2) and (3) is found.  $n_s^*$  becomes an upper bound on  $n_1$  for the double sampling plan and  $c^*$  becomes a lower bound on  $c_2$  where  $c_1$  must be less than  $c_2$ .

The logic behind these limits is that, as  $c_1$  approaches  $c^*$ ,  $n_1$  will approach  $n_s^*$ . If  $c_1$  equals  $c^*$ ,  $n_1$  will equal  $n_s^*$ ,  $n_2$  will approach zero and  $c_2$  will equal  $c_1$ . Thus any derived double sampling plan will degenerate to the corresponding single sampling plan.

The algorithm for deriving the double sampling plan employs equations (2) and (3) replacing the  $L(p)$  function with equation (1) or its Poisson or hypergeometric equivalents. This study concentrates on the binomial of equation (1) only. Initially  $c_2$  is set equal to  $c^*$  and  $c_1$  is varied from 0 to  $c_2-1$ . On each iteration of  $c_1$ ,  $n_1$  is incremented from  $c_1+1$  until the equation

$$B(c_1|n_1, p_2) < \beta$$

is just satisfied. This value of  $n_1$  becomes  $n_{1l}$ , the lower feasible limit on  $n_1$ .

The algorithm then switches to the complete likelihood function, equation (1), searching to satisfy

$$L(p_1) > 1-\alpha$$

$$L(p_2) < \beta.$$

The computational procedure incorporates an integer bisection method in order increase efficiency.

An upper bound on  $n_1$ ,  $n_{1u}$ , is obtained for the current value of  $c_1$  from:

$$B(c_2|n_1, p_2) > 1-\alpha$$

with the constraint:

$$n_{1l} < n_{1u}.$$

The algorithm then switches to the complete likelihood function, equation (1), searching to satisfy:

$$n_{2L}: B(c_1|n_1, p_2) + BB(c_1, c_2|n_1, n_2, p_2) < \beta$$

$$n_{2U}: B(c_1|n_1, p_1) + BB(c_1, c_2|n_1, n_2, p_1) > 1-\alpha$$

$$\text{with } n_{2L} < n_2 < n_{2U}.$$

The lowest possible value for  $n_2$  is  $n_5^* - n_1$ . Since it is necessary to set an upper bound on  $n_2$ ,  $n_{2U}$ , in order to use a bisection search procedure, an arbitrary value of 1.5 times  $n_1$  was used. This value worked successfully in all cases examined herein.

Once the bounds for  $n_2$  have been set, the values of the maximum ASN (ASNMAX) and of the ASN evaluated at  $p_1$  (ASN( $p_1$ )) are calculated for each feasible value of  $n_2$ . These values along with  $n_1$ ,  $n_2$ s and  $n_{2L}$  form the output of the computer program.

#### ANALYSIS AND CONDITIONS FOR OPTIMALITY

The double sampling plan parameters which may be varied are  $n_1$ ,  $n_2$ ,  $c_1$ ,  $c_2$ , and  $r_1$ , five in all. Once minimum values for  $c_1$  and  $c_2$  have been established minimally satisfying the  $(p_1, 1-\alpha)$  and  $(p_2, \beta)$  points on the OC curve, a number of combinations of  $n_1$  and  $n_2$  values also will satisfy these constraints. Furthermore, it is possible to find feasible ranges of  $n_1$  and  $n_2$  combinations for any values of  $c_1$  and  $c_2$  greater than the minimum  $(c_1, c_2)$  combination. Thus an infinite number of plans may be found satisfying the two OC curve points. They represent, in effect, the solution of a pair of algebraic equations in five unknowns.

By introducing the MAXASN constraint, the number of feasible solutions is reduced as it is if the Dodge-Romig scheme of setting  $r_1 = r_2 = c_2 + 1$  is employed. It is the introduction of the objective of minimizing the ASN at the AQL, designated  $p_1$ , that makes it possible to select a single plan and allow the computer program to terminate. In order to accomplish this, the performance of the ASN ( $p_1$ ) was analyzed by varying individually each of the main plan parameters  $n_1$ ,  $n_2$ ,  $c_1$ , and  $c_2$  with  $r_1$  set equal to  $r_2 = c_2 + 1$ .

Examples of this procedure are presented in Appendix I, SAMPLE PROGRAM RUNS. Values entered in each block (cell) are  $ASN(p_1)$ ,  $ASNMAX$ ,  $n_1$  and  $n_2$ . Each block represents the results for a  $c_1$ ,  $c_2$  combination in the feasible region.

#### Some General Constraints

The value of  $c_1$  does not exceed the  $c^*$  of the single sampling plan with the same OC curve. If  $c_1$  equals  $c^*$ , the double sampling plan will degenerate to the single sampling plan with  $n_1 = ns^*$ ,  $n_2=0$ , and  $c_2=c_1$ . Additionally, it is obvious from the ASN formula that, if  $c_1$  is greater than  $c^*$ , the  $ASN(p_1)$  will be greater than  $ns^*$  as will the  $ASNMAX$ . Hence the search on  $c_1$  may be truncated at  $c^*-1$ .

#### Relationship of $\alpha$ , $\beta$ , $p_1$ , and $p_2$

For fixed  $\alpha$  and  $\beta$ , the acceptance number of a single sampling plan will remain constant for a constant discrimination ratio,  $D$ , defined as the ratio  $p_2/p_1$ . As  $p_1$  increases, the sample size of a single sampling plan decreases considerably. This can be explained by the fact that it is the absolute difference ( $p_2-p_1$ ) that influences  $n$ , not the discrimination ratio. For example, for  $p_1=0.02$  and  $p_2=0.10$  the single sample size is approximately 38

with an acceptance number of 3. For  $p_1=0.001$  and  $p_2=0.005$ , the same discrimination ratio, the single sample size is approximately 1,350 with an acceptance number of 3. This confirms that if  $(p_2-p_1)$  is small,  $n$  will be large and if  $(p_2-p_1)$  is large  $n$  will be small. Furthermore as the discrimination ratio decreases to 1.5, the acceptance number increases rather dramatically to 52. This result is verified by use of the Poisson approximation to the binomial and the methodology used to solve for single sampling plans therewith. See Cameron (1952).

Larger values of  $c_1$  result in a smaller range between  $n_{ls}$  and  $n_{ll}$ . As  $c_1$  increases for a specific value of  $c_2$ , the lower bound on  $n_1$  increases. This in turn reduces the number of double sampling plans computed in each cell because the upper bound on  $n_1$ ,  $n_{lu}$ , is dependent on  $c_2$ , not on  $c_1$ .

#### Effect of ASNMAX Constraint, MODEL I

When the constraint  $ASNMAX < ns^*$  is imposed, the values of  $ASN(p_1)$  in the region where  $c_2$  exceeds  $c^*$  become infeasible. An advantage of this property is that the search routine to locate the global minimum does not need to search the region where  $c_2$  exceeds  $c^*$ . However, MODEL II, in which the  $ASNMAX$  constraint is not imposed, requires the evaluation of columns for  $c_2 > c^*$ . It is practically important to study the difference in minimum  $ASN(p_1)$  in each case until a global minimum has been found. Thus, MODEL II searches for column minimums and selects the global minimum from that group. MODEL I needs only to look at the values for  $c_2=c^*$ .

#### Behavior of $ASN(p_1)$ as a Function of $n_1$

Plotting of the values of  $ASN(p_1)$  as a function of  $n_1$  showed that it is a quasi-convex function of  $n_1$ . (Integerization of  $n_1$  and  $n_2$  may explain why the



results were not purely convex) Thus a search for the minimum  $ASN(p_1)$  needs only to continue one step beyond the point at which the minimum exists.

Behavior of  $ASN(p_1)$  as a Function of  $c_1$ .

In general, the  $ASN(p_1)$  proved to be a quasi-convex function of  $c_1$ .

For a constant discrimination ratio, ( $D = p_2/p_1$ ), MODEL I results showed that the minimum  $ASN(p_1)$  occurs at the same value of  $c_1$  irrespective of the value of  $p_1$ . Such is not the case with MODEL II.

The values of  $c_1$  where the minimum  $ASN(p_1)$  occurs is influenced by the ratio  $p_2/p_1$ . As the ratio decreases, the value of  $c_1$  increases. However this relationship also is affected by the magnitude of  $(p_2 - p_1)$ .

Table I.

DESCRIMINATION RATIO ( $P_2/P_1$ )	VALUE of $c_1$ AT min. $ASN(p_1)$ of column. ( $c_2=c^*$ )
25	0
10	0 or 1
5	0 or 1
4	1
3	2
2.5	2 or 3

Table I indicates that as the ratio,  $D$ , decrease, the minimum  $ASN(p_1)$  tends to increase. That is, the corresponding value of  $c_1$  becomes larger. As the ratio increases curves of  $ASN(p_1)$  shift and become truncated constantly increasing from the first feasible solution rather than moving downward to a minimum before sweeping upward. In MODEL II, as  $c_2$  increases, the value  $c_1$  for the minimum  $ASN(p_1)$  of each column may vary.

### Behavior of $ASN(p_1)$ as a Function of $c_2$

Plotting of the  $ASN(p_1)$  values as a function of  $c_2$  yielded quasi-convex results as well. However, there were substantially different results under the two models. When MODEL I (constrained  $ASNMAX$ ) was employed, the minimum  $ASN(p_1)$  value occurred always for values of  $c_2$  equal to  $c^*$ . Under MODEL II, this was the case occasionally but not always.

### Impact of the $ASNMAX$ Constraint

The main objective of the analysis of MODEL II was to evaluate the effect of the constraint  $ASNMAX \leq ns^*$  on the objective function.

Table II shows the minimum  $ASN(p_1)$  obtained from both models together with their  $ASNMAX$  values for some representative cases.

TABLE II

VALUE OF $p_1$	VALUE OF $p_2$	RATIO OF $p_2/p_1$	MODEL I		MODEL II		% OF REDUCTION
			$ASN(p_1)$	$ASN$ MAX	$ASN(p_1)$	$ASN$ MAX	
0.001	0.04	4	179.8	185.	149.1	232.	17.1
0.02	0.8	4	75.4	86.5	73.2	115.	2.23
0.005	0.125	25	19.9	27.1	19.9	27.1	0
0.005	0.05	10	67.1	98.9	67.1	98.9	0
0.04	0.2	5	21.6	31.5	21.6	31.5	0

As indicated, a reduction in  $ASN(p_1)$  generally is obtained only when the  $D$  ratio is very low and/or the difference between  $p_2$  and  $p_1$  is small. For a  $D$  ratio of 4 and  $(p_2 - p_1)$  equals 0.03, a reduction in minimum  $ASN(p_1)$  of 17.1% is obtained by dropping the constraint. However, when the difference between  $p_2$  and  $p_1$  is increased to 0.6 with the same  $D$ , a reduction of only 2.9% in

ASN( $p_1$ ) is seen, but seen at a cost of a substantial increase in ASNMAX. In the rest of the cases illustrated, no reduction in minimum ASN( $p_1$ ) is obtained by eliminating the ASNMAX constraint. For these cases the D ratio was 5 or greater and ( $p_2 - p_1$ ) was 0.045 or greater. These results indicate that both the D ratio and the difference ( $p_2 - p_1$ ) affect ASNMAX but only when both are small.

Additionally, whenever a reduction in minimum ASN( $p_1$ ) is obtained by dropping the constraint, the increase in corresponding ASNMAX value may be substantial. However the increase in ASNMAX may become smaller as the difference between  $p_2$  and  $p_1$  increases. In other words, as the difference between  $p_2$  and  $p_1$  decreases, the price to be paid for the protection against high ASNMAX's will start to increase.

#### SUMMARY OF THE ALGORITHMS

##### MODEL I

- STEP 1. Compute smallest single sampling plan, ( $ns^*$ ,  $c^*$ ).
- STEP 2. Set  $c_2 = c^*$ .
- STEP 3. Incrementing on  $c_1(0, 1, 2, \dots, c^* - 1)$ :
  - 3a. Compute feasible bounds on  $n_1$ ; i.e.,  $n_{1l}$  and  $n_{1u}$ .
  - 3b. Incrementing on  $n_1(n_{1l}, n_{1l} + 1, \dots, n_{1u})$  compute bounds on  $n_2$ ; i.e.,  $n_{2l}$  and  $n_{2u}$ .
  - 3c. Incrementing on  $n_2(n_{2l}, n_{2l} + 1, \dots, n_{2u})$  compute ASNMAX and ASN( $p_1$ ).

##### Condition for Optimality

Feasible values are those for which the likelihood constraints, ( $p_1, 1 - \alpha$ ) and ( $p_2, \beta$ ), and the ASNMAX constraints are satisfied. At each calculation in the feasible region, ASN( $p_1$ ) is calculated and the optimal double sampling

plan is the one with  $\min \text{ASN}(p_1)$ . Because of convexity of  $\text{ASN}(p_1)$ , the algorithm shifts from cell to cell (value of  $c_1$ ) whenever the current calculation ( $\text{ASN}(p_1)$ 's) exceeds that for the previous calculation.

## MODEL II

- STEP 1. Compute the smallest single sampling plan ( $ns^*$ ,  $c^*$ ).
- STEP 2 Incrementing on  $c_2(c^*, c^*+1, c^*+2, \dots)$ :
- STEP 3 Then incrementing on  $c_1(0, 1, 2, \dots, c^*-1)$ :
  - 3a. Compute feasible bounds on  $n_1$ : i.e;  $n_{1l}$  and  $n_{1u}$ .
  - 3b. Incrementing on  $n_1(n_{1l}, n_{1l}+1, \dots, n_{1u})$  compute bounds on  $n_2$ : i.e;  $n_{2l}$  and  $n_{2u}$ .
  - 3c. Incrementing on  $n_2(n_{2l}, n_{2l}+1, n_{2l}+2, \dots, n_{2u})$  compute  $\text{ASN}(p_1)$ .

## Condition for Optimality

Feasible values are those for which the likelihood constraints are satisfied. At each calculation in the feasible region,  $\text{ASN}(p_1)$  is calculated and the optimal double sampling plan is the one with  $\min \text{ASN}(p_1)$ . Because of convexity of  $\text{ASN}(p_1)$  the algorithm shifts from cell to cell (value of  $c_1$ ) until the current calculation ( $\text{ASN}(p_1)$ 's) exceeds that for the previous cell calculation. Similarly the algorithm shifts column to column (value of  $c_2$ ) until the minimum  $\text{ASN}(p_1)$  of the current column exceeds that for the previous column.

## COMPUTER CODE

The program originally was written in FORTRAN IV to run on a PDP 11-34 computer. Later it was modified to allow  $r_1$  to be entered externally (rather than set at  $r_2 = c_2+1$ ) and to run on a VAX 11-750 computer. The complete code is included in APPENDIX II.

As stated previously, the single sampling plan is computed using a brute force method; i.e: the search starts with an acceptance number of zero and the sample size is incremented by one at each iteration until  $L(p_1)$  and  $L(p_2)$  satisfy their respective inequalities. If the solved value of  $n_1$  exceeds  $n_u$ , no feasible solution exists for that value of  $c$ ,  $c$  is incremented by one, and the search process for  $n_1$  and  $n_u$  starts anew. Depending on the input parameters ( $\alpha$ ,  $\beta$ ,  $p_1$  &  $p_2$ ), the single sample size may become very large thus requiring a large number of iterations to reach the first feasible solution. To reduce unnecessary computations, the user may input a "seed" number as a starting value of the single sample size. The closer the seed is to the true solution, the lesser the number of iterations required. However, the user must be very careful in entering a seed value. If a higher value of the seed than the true solution minimum  $n_s$  is entered, the algorithm will converge to a single sampling plan with an acceptance number higher than that of minimum single sampling plan (the desired solution).

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APPENDIX I  
SAMPLE PROGRAM RUNS

DEPT. OF ISE  
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\*\*\*\*\*DOUBLE SAMPLING SYSTEM\*\*\*\*\*

ALPHA =0.0500      BETA =0.1000  
PO =0.0100      P1 =0.0400

REJECTION NO. OF FIRST SAMPLE (R1) = C2+( 1)

S-----  
ACCEPTANCE NO. (C) = 4  
LOWER BOUND ON N (NS) = 198  
UPPER BOUND ON N (NL) = 198

DOUBLE SAMPLING PLANS

FOR C1= 0 C2= 4

136	62	62	186.8584	181.4362
137	61	61	187.0367	181.8368

DOUBLE SAMPLING PLANS

FOR C1= 1 C2= 4

164	34	34	185.1312	179.7643
165	33	33	185.5088	180.3866

DOUBLE SAMPLING PLANS

FOR C1= 2 C2= 4

183	15	15	189.1240	186.5909
184	14	14	189.7155	187.3789

DOUBLE SAMPLING PLANS

FOR C1= 3 C2= 4

188	11	11	190.1723	188.8725
194	4	4	194.7896	194.3390



S-----  
 ACCEPTANCE NO. (C) = 5  
 LOWER BOUND ON N (NS) = 230  
 UPPER BOUND ON N (NL) = 262

DOUBLE SAMPLING PLANS

FOR C1= 0 C2= 5

63	213	213	251.3808	162.9084
64	207	212	247.0489	162.1922
65	203	210	244.4871	162.3609

DOUBLE SAMPLING PLANS

FOR C1= 1 C2= 5

101	180	183	232.8200	149.1270
102	174	181	229.4124	149.1608

DOUBLE SAMPLING PLANS

FOR C1= 2 C2= 5

134	161	162	223.2362	157.9882
135	151	160	218.6858	157.8415
136	143	158	215.2449	157.9577

DOUBLE SAMPLING PLANS

FOR C1= 3 C2= 5

166	149	155	220.4352	177.8430
167	127	151	213.3944	177.2542
168	114	148	209.6420	177.3490

DOUBLE SAMPLING PLANS

FOR C1= 4 C2= 5

198	134	262	221.8151	202.6424
199	87	262	214.4609	202.0609
200	72	262	212.7945	202.5721

GLOBAL MINIMUM ASN(PO)= 149.16

CORRESPONDING N1 = 102

CORRESPONDING N2S = 174

CORRESPONDING C1 = 1

CORRESPONDING C2 = 5

DEPT. OF ISE  
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\*\*\*\*\*DOUBLE SAMPLING SYSTEM\*\*\*\*\*

ALPHA =0.0500      BETA =0.1000  
PO =0.0200      P1 =0.0800

REJECTION NO. OF FIRST SAMPLE (R1) = C2+( 1)

S-----  
ACCEPTANCE NO. (C) = 4  
LOWER BOUND ON N (NS) = 98  
UPPER BOUND ON N (NL) = 99

DOUBLE SAMPLING PLANS

FOR C1= 0 C2= 4

46	54	54	90.6816	78.5600
47	53	53	90.8418	79.3637

DOUBLE SAMPLING PLANS

FOR C1= 1 C2= 4

62	39	39	86.5216	75.4389
63	38	38	86.8856	76.3481

DOUBLE SAMPLING PLANS

FOR C1= 2 C2= 4

75	27	27	86.1676	79.6567
77	24	24	86.9210	81.3386

DOUBLE SAMPLING PLANS

FOR C1= 3 C2= 4

87	16	16	90.2002	88.0654
88	14	15	90.7994	88.9571

S-----  
 ACCEPTANCE NO. (C) = 5  
 LOWER BOUND ON N (NS) = 114  
 UPPER BOUND ON N (NL) = 131

DOUBLE SAMPLING PLANS

FOR C1= 0 C2= 5

31	105	108	124. 7199	79. 8665
32	100	106	121. 2064	79. 6080
33	96	105	118. 5924	79. 7088

DOUBLE SAMPLING PLANS

FOR C1= 1 C2= 5

50	88	93	115. 1913	73. 2101
51	82	91	111. 7191	73. 2325

DOUBLE SAMPLING PLANS

FOR C1= 2 C2= 5

66	81	84	111. 4853	77. 6491
67	72	81	107. 4158	77. 6818

DOUBLE SAMPLING PLANS

FOR C1= 3 C2= 5

82	75	83	109. 7731	87. 7234
83	57	79	104. 1007	87. 4925
84	49	75	102. 1335	87. 9863

DOUBLE SAMPLING PLANS

FOR C1= 4 C2= 5

98	63	131	109. 3472	100. 0915
99	35	131	105. 3023	100. 1993

GLOBAL MINIMUM ASN(PO)= 73. 23

CORRESPONDING N1 = 51

CORRESPONDING N2S = 82

CORRESPONDING C1 = 1

CORRESPONDING C2 = 5

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\*\*\*\*\*DOUBLE SAMPLING SYSTEM\*\*\*\*\*

ALPHA =0.0500      BETA =0.1000  
P0 =0.0200      P1 =0.0600

REJECTION NO. OF FIRST SAMPLE (R1) = C2+( 1)

S-----  
ACCEPTANCE NO. (C) = 7  
LOWER BOUND ON N (NS) = 194  
UPPER BOUND ON N (NL) = 200

DOUBLE SAMPLING PLANS

FOR C1= 0 C2= 7

54	149	149	195.7031	152.9495
55	147	148	194.7807	153.6086

DOUBLE SAMPLING PLANS

FOR C1= 1 C2= 7

80	124	124	187.7091	139.1258
81	122	123	186.9561	139.9612

DOUBLE SAMPLING PLANS

FOR C1= 2 C2= 7

101	105	105	180.3470	135.4178
102	103	103	179.8239	136.3186

DOUBLE SAMPLING PLANS

FOR C1= 3 C2= 7

121	86	86	174.2581	140.0145
122	84	85	174.0120	140.9156

DOUBLE SAMPLING PLANS

FOR C1= 4 C2= 7

139	71	71	172.2300	148.9510
140	69	69	172.2892	149.8648

DOUBLE SAMPLING PLANS

FOR C1= 5 C2= 7

157	57	57	174.6661	161.6958
158	53	55	174.4244	162.4568

DOUBLE SAMPLING PLANS

FOR C1= 6 C2= 7

175	43	51	181.5391	176.6320
176	36	47	181.4739	177.3945

-----  
 ACCEPTANCE NO. (C) = 8  
 LOWER BOUND ON N (NS) = 215  
 UPPER BOUND ON N (NL) = 236

DOUBLE SAMPLING PLANS

FOR C1= 0 C2= 8

44	197	199	234. 9933	160. 0134
45	193	198	232. 0821	160. 2446

DOUBLE SAMPLING PLANS

FOR C1= 1 C2= 8

70	173	176	227. 3926	140. 8520
71	169	174	224. 7274	141. 3875

DOUBLE SAMPLING PLANS

FOR C1= 2 C2= 8

92	156	157	220. 2482	135. 6124
93	151	156	217. 1164	136. 0340

DOUBLE SAMPLING PLANS

FOR C1= 3 C2= 8

113	140	141	212. 4149	139. 6817
114	134	139	209. 1393	140. 0779

DOUBLE SAMPLING PLANS

FOR C1= 4 C2= 8

134	120	125	203. 5589	149. 6677
135	113	123	200. 4914	150. 0802

DOUBLE SAMPLING PLANS

FOR C1= 5 C2= 8

154	106	116	200. 3735	163. 1247
155	95	113	196. 5552	163. 3638

DOUBLE SAMPLING PLANS

FOR C1= 6 C2= 8

174	91	125	200. 3812	178. 8247
175	75	118	196. 7400	179. 0665

DOUBLE SAMPLING PLANS

FOR C1= 7 C2= 8

194	76	236	204. 8341	195. 9521
195	49	236	201. 9844	196. 2862

-----  
 ACCEPTANCE NO. (C) = 9  
 LOWER BOUND ON N (NS) = 235  
 UPPER BOUND ON N (NL) = 273

DOUBLE SAMPLING PLANS

FOR C1= 0 C2= 9

41	236	242	272. 4937	173. 9185
42	229	240	266. 5910	172. 9765
43	224	239	262. 6524	173. 0344

DOUBLE SAMPLING PLANS

FOR C1= 1 C2= 9

66	221	222	273. 3678	150. 2853
67	212	220	265. 8895	149. 3582
68	205	218	260. 2918	149. 0869
69	200	217	256. 5717	149. 5144

DOUBLE SAMPLING PLANS

FOR C1= 2 C2= 9

89	204	204	266. 7397	142. 7357
90	193	202	258. 1279	141. 8807
91	185	200	252. 1336	141. 7317
92	179	198	247. 8829	142. 0573

DOUBLE SAMPLING PLANS

FOR C1= 3 C2= 9

111	186	188	256. 2776	145. 0258
112	173	186	247. 1019	144. 3393
113	164	184	241. 0539	144. 3170
114	157	182	236. 5696	144. 6165

DOUBLE SAMPLING PLANS

FOR C1= 4 C2= 9

132	175	176	249. 4611	154. 0411
133	156	174	237. 6933	153. 0990
134	145	171	231. 2966	153. 1052

DOUBLE SAMPLING PLANS

FOR C1= 5 C2= 9

153	154	171	237. 1488	166. 3868
154	132	167	226. 1181	165. 7449
155	121	164	221. 0988	166. 0171

# DOUBLE SAMPLING PLANS

FOR C1= 6 C2= 9

174	120	189	223.4432	181.0777
175	103	181	217.4335	181.2191

# DOUBLE SAMPLING PLANS

FOR C1= 7 C2= 9

194	112	273	224.6025	198.0900
195	81	273	217.1298	198.0276
196	70	273	215.1222	198.6775

# DOUBLE SAMPLING PLANS

FOR C1= 8 C2= 9

215	53	273	222.1338	215.9652
216	41	273	221.5181	216.7635

GLOBAL MINIMUM ASN(P0)= 135.42

CORRESPONDING N1 = 101

CORRESPONDING N2S = 105

CORRESPONDING C1 = 2

CORRESPONDING C2 = 7

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\*\*\*\*\*DOUBLE SAMPLING SYSTEM\*\*\*\*\*

ALPHA =0.0500      BETA =0.1000  
PO =0.0150      P1 =0.0450

REJECTION NO. OF FIRST SAMPLE (R1) = C2+( 1)

S-----  
ACCEPTANCE NO. (C) = 7  
LOWER BOUND ON N (NS) = 260  
UPPER BOUND ON N (NL) = 266

DOUBLE SAMPLING PLANS

FOR C1= 0 C2= 7

75	195	195	260.0430	207.2268
76	193	194	259.1314	207.8022

DOUBLE SAMPLING PLANS

FOR C1= 1 C2= 7

108	164	164	250.0084	187.1743
109	162	162	249.2654	187.9875

DOUBLE SAMPLING PLANS

FOR C1= 2 C2= 7

138	135	135	239.5929	184.0606
139	133	134	239.0794	184.9139

DOUBLE SAMPLING PLANS

FOR C1= 3 C2= 7

164	111	111	232.4163	189.4726
165	109	109	232.1787	190.3486

DOUBLE SAMPLING PLANS

FOR C1= 4 C2= 7

188	90	90	229.9069	201.1494
189	87	88	229.5068	201.8962

DOUBLE SAMPLING PLANS

FOR C1= 5 C2= 7

211	72	72	233.1982	217.1050
212	68	70	232.9633	217.8536

DOUBLE SAMPLING PLANS

FOR C1= 6 C2= 7

234	59	64	242.9257	236.2712
235	51	60	242.7148	236.9932



S-----  
 ACCEPTANCE NO. (C) = 8  
 LOWER BOUND ON N (NS) = 28/  
 UPPER BOUND ON N (NL) = 314

DOUBLE SAMPLING PLANS

FOR C1= 0 C2= 8

59	264	265	314. 4307	214. 7717
60	260	263	311. 5352	215. 0103

DOUBLE SAMPLING PLANS

FOR C1= 1 C2= 8

93	235	235	306. 1528	188. 7529
94	231	233	303. 5039	189. 3262

DOUBLE SAMPLING PLANS

FOR C1= 2 C2= 8

124	205	207	291. 8433	182. 4429
125	200	206	288. 7336	182. 8303

DOUBLE SAMPLING PLANS

FOR C1= 3 C2= 8

152	183	185	281. 3353	187. 6786
153	177	183	278. 0829	188. 0451

DOUBLE SAMPLING PLANS

FOR C1= 4 C2= 8

179	164	166	273. 5826	200. 6032
180	156	164	269. 9618	200. 8872

DOUBLE SAMPLING PLANS

FOR C1= 5 C2= 8

206	143	151	268. 2267	218. 5081
207	131	148	264. 0007	218. 6513

DOUBLE SAMPLING PLANS

FOR C1= 6 C2= 8

232	157	163	277. 2705	240. 3717
233	117	157	266. 7340	239. 3440
234	101	151	263. 1187	239. 5680

DOUBLE SAMPLING PLANS

FOR C1= 7 C2= 8

260	80	314	271. 3427	262. 1125
261	63	314	269. 9318	262. 6904

S-----  
 ACCEPTANCE NO. (C) = 9  
 LOWER BOUND ON N (NS) = 314  
 UPPER BOUND ON N (NL) = 363

# DOUBLE SAMPLING PLANS

FOR C1= 0 C2= 9

55	316	322	364. 4188	233. 3808
56	309	320	358. 5363	232. 4478
57	303	319	353. 6355	231. 9713
58	298	317	349. 7147	231. 9728

# DOUBLE SAMPLING PLANS

FOR C1= 1 C2= 9

89	291	294	361. 2587	201. 4461
90	283	292	354. 7504	200. 8535
91	276	290	349. 1775	200. 5675
92	270	289	344. 5428	200. 6042

# DOUBLE SAMPLING PLANS

FOR C1= 2 C2= 9

120	264	269	349. 1373	191. 0035
121	255	267	342. 3050	190. 6154
122	247	265	336. 3438	190. 4330
123	241	263	332. 1181	190. 7493

# DOUBLE SAMPLING PLANS

FOR C1= 3 C2= 9

149	243	248	337. 9465	194. 2755
150	231	246	329. 6002	193. 7339
151	222	244	323. 5883	193. 7005
152	215	242	319. 1306	194. 0067

# DOUBLE SAMPLING PLANS

FOR C1= 4 C2= 9

177	226	232	327. 9336	206. 0636
178	209	229	317. 5687	205. 3329
179	197	227	310. 5440	205. 1960
180	189	224	306. 1929	205. 5502

# DOUBLE SAMPLING PLANS

FOR C1= 5 C2= 9

205	198	223	312. 6136	222. 6100
206	178	220	302. 7364	222. 1069
207	166	216	297. 2081	222. 2803

# DOUBLE SAMPLING PLANS

FOR C1= 6 C2= 9

232	200	248	313. 9585	243. 8767
233	156	241	296. 9233	242. 4273
234	139	233	290. 9528	242. 5471

DOUBLE SAMPLING PLANS

FOR C1= 7 C2= 9

260	126	363	294.2362	264.7458
261	106	363	289.8002	265.0618

DOUBLE SAMPLING PLANS

FOR C1= 8 C2= 9

287	82	363	297.9774	288.5138
288	63	363	296.4333	289.1825

GLOBAL MINIMUM ASN(PO)= 184.06

CORRESPONDING N1 = 138

CORRESPONDING N2S = 135

CORRESPONDING C1 = 2

CORRESPONDING C2 = 7

APPENDIX II  
FORTRAN IV PROGRAM

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0001 C QUALITY CONTROL DOUBLE SAMPLING PROGRAM TO ANALYSE
0002 C DOUBLE SAMPLING PLANS, ASN(P0) AND ASNMAX.
0003 C BINOMIAL AND POISON PROBABILITY DISTRIBUTIONS USED.
0004 C
0005 C PROGRAMED BY R. WARREN RANGARAJAN
0006 C INDUSTRIAL AND SYSTEMS ENGINEERING DEPARTMENT
0007 C UNIVERSITY OF FLORIDA
0008 C GAINESVILLE, FLORIDA 32611
0009 C
0010 C DOUBLE PRECISION SUMLOG
0011 C INTEGER C,C1,C2,C1MIN,C2MIN,R1,R11
0012 C BYTE STING(8)
0013 C COMMON/BLK1/N2S,N2L
0014 C COMMON/BLK2/PS,PL
0015 C COMMON/BLK3/N1
0016 C COMMON/BLK4/ALPHA,BETA
0017 C COMMON/BLK5/P0,P1
0018 C COMMON/BLK6/C1,C2
0019 C COMMON/BLK7/SUMLOG(2500)
0020 C COMMON/BLK8/N
0021 C COMMON/BLK9/C2MAX,C1MAX(15)
0022 C COMMON/BLK10/NS,NL
0023 C COMMON/BLK11/ASN,ASNMAX
0024 C
0025 C
0026 C WRITE(5,*) ' NAME OF OUTPUT FILE?'
0027 C READ(5,1) STING
0028 C 1 FORMAT(10A1)
0029 C
0030 C CALL ASSIGN (1,STING)
0031 C
0032 C BEGINNING INITIALIZATION
0033 C
0034 C N=0
0035 C C2=1000
0036 C ASNMIN=15000.
0037 C C=-1
0038 C
0039 C INPUT FORMAT
0040 C
0041 C 15 WRITE (5,16)
0042 C 16 FORMAT (////' CODES FOR SELECTING APPR. PROB. DIST. '//
0043 C 115X,'BINOMIAL',12X,'=1',
0044 C 2/15X,'POISSON',13X,'=2')
0045 C READ (5,*) K
0046 C IF(K.GT.2.OR.K.LT.1) GOTO 15
0047 C 22 WRITE(5,21)
0048 C 21 FORMAT(10X,'SELECT'/16X,'SAMPLE PLANS ONLY =1'
0049 C 1/16X,'ASN VALUES ONLY =2'
0050 C 2/16X,'OR BOTH =3')
0051 C READ(5,*) KOPT
0052 C IF(KOPT.GT.3.OR.KOPT.LT.1) GOTO 22
0053 C WRITE (5,17)
0054 C 17 FORMAT(10X,'INPUT ALPHA ')
0055 C READ (5,*) ALPHA
0056 C WRITE (5,18)
0057 C 18 FORMAT(10X,'INPUT BETA ')

```

```

0058      READ (5,*) BETA
0059      C
0060      WRITE (5,51)
0061      51 FORMAT(10X, 'INPUT P0 ')
0062      READ (5,*) P0
0063      WRITE(5,52)
0064      52 FORMAT(10X, 'INPUT P1 ')
0065      READ (5,*) P1
0066      WRITE(5,58)
0067      58 FORMAT( 5X, 'INPUT A SEED FOR SINGLE SAMPLING NO. '//
0068      1' IF NO SEED AVAILABLE ENTER ZERO AS THE SEED VALUE')
0069      READ(5,*) NS
0070      WRITE(5,59)
0071      59 FORMAT( 5X, 'INPUT A VALUE FOR (R1-C2) '//
0072      1' IF R1=C2 THEN THE VALUE WOULD BE 0'//
0073      2' IF R1>C2 THEN THE VALUE WOULD BE A POSITIVE NO. '//
0074      3' IF R1<C2 THEN THE VALUE WOULD BE A NEGATIVE NO. ')
0075      READ(5,*) R1
0076      C
0077      MC1=10.0/(P1/P0)
0078      WRITE (1,53)
0079      53 FORMAT(///10X, 'DEPT. OF ISE '
0080      1/, 10X, 'UNIVERSITY OF FLORIDA '
0081      2/5X, 5(' '), 'DOUBLE SAMPLING SYSTEM', 5(' '), 2X, /)
0082      WRITE (5,54) ALPHA, BETA, P0, P1
0083      WRITE (1,54) ALPHA, BETA, P0, P1
0084      54 FORMAT(//10X, 'ALPHA =', F6.4, 5X, 'BETA =', F6.4,
0085      1/10X, 'P0 =', F6.4, 8X, 'P1 =', F6.4)
0086      WRITE(5,55) R1
0087      WRITE(1,55) R1
0088      55 FORMAT(/5X, 'REJECTION NO. OF FIRST SAMPLE (R1) = C2+(', I3, ')')
0089      5 C=C+1
0090      C
0091      C
0092      C
0093      C
0094      C
0095      C
0096      C
0097      C
0098      C
0099      C
0100      C
0101      C
0102      C
0103      C
0104      C
0105      C
0106      C
0107      C
0108      C
0109      C
0110      C
0111      C
0112      C
0113      C
0114      C

```

READ (5,\*) BETA  
 WRITE (5,51)  
 51 FORMAT(10X, 'INPUT P0 ')  
 READ (5,\*) P0  
 WRITE(5,52)  
 52 FORMAT(10X, 'INPUT P1 ')  
 READ (5,\*) P1  
 WRITE(5,58)  
 58 FORMAT( 5X, 'INPUT A SEED FOR SINGLE SAMPLING NO. '//  
 1' IF NO SEED AVAILABLE ENTER ZERO AS THE SEED VALUE')  
 READ(5,\*) NS  
 WRITE(5,59)  
 59 FORMAT( 5X, 'INPUT A VALUE FOR (R1-C2) '//  
 1' IF R1=C2 THEN THE VALUE WOULD BE 0'//  
 2' IF R1>C2 THEN THE VALUE WOULD BE A POSITIVE NO. '//  
 3' IF R1<C2 THEN THE VALUE WOULD BE A NEGATIVE NO. ')  
 READ(5,\*) R1  
 MC1=10.0/(P1/P0)  
 WRITE (1,53)  
 53 FORMAT(///10X, 'DEPT. OF ISE '  
 1/, 10X, 'UNIVERSITY OF FLORIDA '  
 2/5X, 5(' '), 'DOUBLE SAMPLING SYSTEM', 5(' '), 2X, /)  
 WRITE (5,54) ALPHA, BETA, P0, P1  
 WRITE (1,54) ALPHA, BETA, P0, P1  
 54 FORMAT(//10X, 'ALPHA =', F6.4, 5X, 'BETA =', F6.4,  
 1/10X, 'P0 =', F6.4, 8X, 'P1 =', F6.4)  
 WRITE(5,55) R1  
 WRITE(1,55) R1  
 55 FORMAT(/5X, 'REJECTION NO. OF FIRST SAMPLE (R1) = C2+(', I3, ')')  
 5 C=C+1  
 SINGLE SAMPLING PROCEDURE BEGINS  
 KK1=C+1  
 10 NS=NS+1  
 COMPUTATION OF LOWER BOUND OF SINGLE SAMPLING PLAN  
 IF(K.EQ.1) CALL PROBS1(NS,P1,C,BXLEC,N)  
 IF(K.EQ.2) CALL PROBS2(NS,P1,C,BXLEC,N)  
 IF(BXLEC.GT.BETA) GOTO 10  
 NLT=NS-5  
 NL=MAX0(1,NLT)  
 COMPUTATION OF UPPER BOUND OF SINGLE SAMPLING PLAN  
 20 NL=NL+1  
 IF(K.EQ.1) CALL PROBS1(NL,P0,C,BXLEC)  
 IF(K.EQ.2) CALL PROBS2(NL,P0,C,BXLEC)  
 IF(BXLEC.GE.(1-ALPHA)) GOTO 20  
 NL=NL-1  
 TEST FOR FEASIBILITY  
 IF(NS.GT.NL) GOTO 5

```

0115      WRITE (5,90)
0116      WRITE(1,90)
0117      90 FORMAT(/10X, 'SINGLE SAMPLING PLAN '//+',',9X,21('...'))
0118      WRITE(5,91) C,NS,NL
0119      WRITE(1,91) C,NS,NL
0120      91 FORMAT(10X, 'ACCEPTANCE NO. (C) =', I2
0121      1, /10X, 'LOWER BOUND ON N (NS) =', I4
0122      2, /10X, 'UPPER BOUND ON N (NL) =', I4)
0123      C
0124      C      COMPUTATION OF DOUBLE SAMPLING PLAN BEGINS: FOR EACH VALUE OF C2
0125      C
0126      IF(C.LT.C2) MC=C+MC1-1
0127      C2=C
0128      C
0129      R1=C2+R11
0130      C
0131      DO 100 K1=1,C2
0132      C1=K1-1
0133      C
0134      C      CALL SUBROUTINE TO COMPUTE THE FIRST SAMPLE NUMBER
0135      C
0136      CALL TRY1(NTRY,C1,P1,NS,BETA,K)
0137      N1=NTRY
0138      IF(NTRY.GT.NS) GOTO 600
0139      C
0140      C
0141      WRITE(5,161)
0142      WRITE(1,161)
0143      161 FORMAT(/10X, 'DOUBLE SAMPLING PLANS',/)
0144      WRITE(5,160) C1,C2
0145      WRITE(1,160) C1,C2
0146      160 FORMAT(/10X, 'FOR C1=', I2, 2X, 'C2=', I2, //)
0147      C
0148      NTEMP=N1
0149      IF(KOPT.EQ.1) WRITE(5,170)
0150      IF(KOPT.EQ.3) WRITE(5,175)
0151      175 FORMAT(10X, '(N1)', 3X, '(N2S)', 3X, '(N2L)', 4X, 'ASNMAX', 5X, 'ASN'
0152      1
0153      170 FORMAT(11X, '(N1)', 10X, '(N2S) ( N2 ( (N2L)', 8X, 'PS',
0154      1
0155      10X, 'PL'//)
0156      NTEMP1=NS
0157      C
0158      C      COMPUTATION OF SECOND SAMPLE FOR EACH VALUE OF FIRST SAMPLE
0159      C
0160      ASN=FLOAT(NS)*10
0161      DO 190 IZ=NTEMP,NTEMP1
0162      I=IZ
0163      C
0164      C      CALL SUBROUTINE TO COMPUTE SECOND SAMPLE
0165      C
0166      CALL TRY2(NS,NL,K,I,R1)
0167      IF(KOPT.NE.1) GOTO 500
0168      WRITE (5,185) I,N2S,N2L,PS,PL
0169      WRITE(1,185) I,N2S,N2L,PS,PL
0170      185 FORMAT(10X, I4, 13X, I4, ' ( N2 ( ', I4, 4X, F8.6, 8X, F8.6)
0171      GOTO 190
0172      C

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JQCDS7\$MAIN

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0172 C      TEST FOR FEASIBILITY
0173 C
0174      500      IF(N2S.GT.N2L) GOTO 190
0175      IF(N2S.LT.(C2-C1).OR.I.LE.C2) GOTO 190
0176      ASNTEM=ASN
0177 C
0178 C      CALL SUBROUTINE TO COMPUTE ASN(PO) AND ASNMAX VALUES
0179 C
0180      CALL ASNN(MC,NS,K,I,KOPT,C1MIN,C2MIN,N1MIN,N2MIN,ASNMIN)
0181      IF(KOPT.NE.3) GOTO 190
0182      WRITE(5,410)I,N2S,N2L,ASNMAX,ASN
0183      WRITE(1,410)I,N2S,N2L,ASNMAX,ASN
0184      410      FORMAT(10X,I3,4X,I3,5X,I3,5X,2(F8.4,3X))
0185      IF(ASN.GT.ASNTEM) GOTO 100
0186      190      CONTINUE
0187 C
0188 C
0189      100 CONTINUE
0190 C
0191      600 IF(C.LT.MC) GOTO 5
0192      WRITE(1,601) ASNMIN,N1MIN,N2MIN,C1MIN,C2MIN
0193      WRITE(5,601) ASNMIN,N1MIN,N2MIN,C1MIN,C2MIN
0194      601 FORMAT( 10X,'GLOBAL MINIMUM ASN(PO)=' ,F8.2, //
0195      110X,'CORRESPONDING N1      =' ,I5//
0196      210X,'CORRESPONDING N2S     =' ,I5//
0197      310X,'CORRESPONDING C1      =' ,I2//
0198      410X,'CORRESPONDING C2      =' ,I2)
0199 C
0200 C
0201      STOP
0202      END

```



```

0001      C
0002      C
0003      C
0004      C
0005      SUBROUTINE TRY1(NTRY,C1,P,NL,BETA,K)
0006      C
0007      THIS SUBROUTINE COMPUTES FIRST SAMPLE NUMBER OF DOUBLE
0008      SAMPLING PLAN BY AN INTEGER FORM OF BISECTION METHOD
0009      C
0010      INTEGER C1
0011      C
0012      C
0013      NLARGE=NL
0014      NSMALL=0
0015      C
0016      5 NTRY=(NSMALL+NLARGE)/2.0
0017      C  CALL APPROPRIATE PROBAILITY SUBROUTINE FOR PROB. CALCULATIONS
0018      10 IF(K.EQ.1) CALL PROBS1(NTRY,P,C1,BXLEC)
0019      IF(K.EQ.2) CALL PROBS2(NTRY,P,C1,BXLEC)
0020      IF(BXLEC.LE.BETA) GOTO 50
0021      NSMALL=NTRY
0022      GOTO 25
0023      50 NLARGE=NTRY
0024      25 IF(NSMALL.NE.(NLARGE-1)) GOTO 5
0025      NTRY=NLARGE
0026      RETURN
0027      END

```

```

0001      C
0002      C
0003      C
0004      C
0005      SUBROUTINE PROBD1(N1,N2,P,DPROB,K,R1)
0006      C
0007      THIS SUBROUTINE COMPUTES DOUBLE PROBABILITIES FOR
0008      COMPUTING SECOND SAMPLE NUMBER OF DOUBLE SAMPLING NUMBER
0009      C
0010      COMMON/BLK6/C1,C2
0011      INTEGER C1,C2,R1
0012      C
0013      C
0014      IF(K.EQ.1) CALL PROBS1(N1,P,C1,BXLEC)
0015      IF(K.EQ.2) CALL PROBS2(N1,P,C1,BXLEC)
0016      DPROB=BXLEC
0017      TEMP=BXLEC
0018      NTEMP=C1+1
0019      KTEMP=R1-1
0020      DO 10 IX=NTEMP,KTEMP
0021          I=IX
0022          J=C2-I
0023          IF(K.EQ.1) CALL PROBS1(N1,P,I,BXLEC)
0024          IF(K.EQ.2) CALL PROBS2(N1,P,I,BXLEC)
0025          PROB1=BXLEC-TEMP
0026          TEMP=BXLEC
0027          IF(K.EQ.1) CALL PROBS1(N2,P,J,BXLEC)
0028          IF(K.EQ.2) CALL PROBS2(N2,P,J,BXLEC)
0029          DPROB=DPROB+(PROB1*BXLEC)
0030      10 CONTINUE
0031      C
0032      RETURN
0033      END

```

```

0001      C
0002      C
0003      C
0004      C
0005      SUBROUTINE TRY2(NS,NL,K,J,R1)
0006      C
0007      C      THIS SUBROUTINE COMPUTES THE SECOND SAMPLE NUMBER OF
0008      C      THE DOUBLE SAMPLING NUMBER BY AN INTEGER BISECTION
0009      C      METHOD. SEVERAL TESTS ARE DONE TO LOCATE THE PARAMETER
0010      C      AT ITS TRUE POSITION.
0011      C
0012      C      INTEGER C1,C2,R1
0013      C
0014      C      COMMON/BLK1/N2S,N2L
0015      C      COMMON/BLK2/PS,PL
0016      C      COMMON/BLK3/N1
0017      C      COMMON/BLK4/ALPHA,BETA
0018      C      COMMON/BLK5/PO,P1
0019      C      COMMON/BLK6/C1,C2
0020      C
0021      C      K1=C1+1
0022      C
0023      C      SET LIMITS FOR COMPUTING N2S
0024      C
0025      C      NSMALL=NS-J
0026      C      NLARGE=NSMALL
0027      C
0028      C      INDEXING TO SPECIFY WHAT BOUND (N2S OR N2L) IS BEING
0029      C      COMPUTED
0030      C
0031      C      I=1
0032      C
0033      C      INITIAL TEST AT EACH LIMIT
0034      C
0035      C      CALL PROBD1(J,NSMALL,P1,DPROB,K,R1)
0036      C      IF(DPROB.LE.BETA) GOTO 55
0037      C
0038      C      BISECTION METHOD
0039      C
0040      C      NLARGE=NL
0041      C      5 NTRY=(NSMALL+NLARGE)/2.0
0042      C      GOTO (10,20),I
0043      C
0044      C      10 CALL PROBD1(J,NTRY,P1,DPROB,K,R1)
0045      C      IF(DPROB.LE.BETA) GOTO 50
0046      C      GOTO 15
0047      C      20 CALL PROBD1(J,NTRY,PO,DPROB,K,R1)
0048      C      IF(DPROB.LT.(1-ALPHA)) GOTO 50
0049      C      15 NSMALL=NTRY
0050      C      GOTO 25
0051      C      50 NLARGE=NTRY
0052      C      25 IF((NLARGE-NSMALL).GT.1) GOTO 5
0053      C
0054      C      CHECK THE INDEX TO FIND WHERE THE PROCESS IS
0055      C
0056      C      GOTO (55,60),I
0057      C

```

TRY2

```
0058 C      CHANGE THE INDEX AFTER N2S COMPUTATION
0059 C
0060      55 I=I+1
0061 C
0062 C      TESTING EACH POSSIBLE CASES TO LOCATE
0063 C      THE LOWER BOUND AT ITS TRUE POSITION
0064 C
0065      N2S=MAX0(0,NLARGE)
0066      CALL PROBD1(J,NLARGE,P1,DPROB,K,R1)
0067      PS=DPROB
0068      MTEMP=NLARGE-5
0069      NSMALL=MAX0(0,MTEMP)
0070      NLARGE=NL
0071      GOTO 5
0072      60 N2L=NSMALL
0073      CALL PROBD1(J,NSMALL,P0,DPROB,K,R1)
0074      PL=DPROB
0075      CALL PROBD1(J,NLARGE,P0,DPROB,K,R1)
0076      IF(DPROB.GE.(1-ALPHA)) N2L=NLARGE
0077      IF(DPROB.GE.(1-ALPHA)) PL=DPROB
0078 C
0079 C
0080      110 RETURN
0081      END
```

```

0001      C
0002      C
0003      C
0004      C
0005      SUBROUTINE PROBS1(NN,P,C,BXLEC)
0006      C
0007      THIS SUBROUTINE COMPUTES CUMULATIVE BINOMIAL
0008      C      PROBABILITIES
0009      C
0010      INTEGER C
0011      DOUBLE PRECISION SUMLOG
0012      COMMON/BLK7/SUMLOG(1500)
0013      COMMON/BLK8/N
0014      C
0015      C
0016      Q=1.-P
0017      C
0018      BINOMIAL PROB. WHEN C=0
0019      C
0020      CSUMS=Q**NN
0021      C      WRITE(6,500) CSUMS
0022      IF (C.EQ.0) GOTO 333
0023      C
0024      AVOID RECOMPUTING SUMLOG(I)'S ALREADY IN MEMORY
0025      C
0026      IF (N-NN) 100,211,211
0027      100 M=N+1
0028      C
0029      COMPUTE N SUMLOGS-EQUIVALENT TO N-FACTORIAL
0030      C
0031      IF (M.GT.1) GOTO 110
0032      SUMLOG(1)=0.
0033      IF(NN.LE.1) GOTO 211
0034      M=2
0035      110 DO 111 I=M,NN
0036          SUMLOG(I)=DLOG10(DFLOAT(I))+SUMLOG(I-1)
0037      111 CONTINUE
0038      C
0039      COMPUTE C SUMS-EQUIVALENT TO SSUM OF PROB.COMPIN.
0040      C      I.E. CUMULATIVE BINOMIAL DISTRIBUTION COMPUTATION
0041      C
0042      211 IF(NN.GT.N) N=NN
0043      C
0044      DETERMINE BEST NUMBER HANDLING LOOP
0045      C
0046      IF (NN.GT.300) GOTO 300
0047      DO 222 K=1,C
0048          CSUMS=10.**((SUMLOG(NN)-SUMLOG(NN-K)-SUMLOG(K))
0049          1          *P**K*Q**(NN-K)+CSUMS)
0050      222 CONTINUE
0051      C      WRITE(6,501) CSUMS
0052      C      501 FORMAT(5X,'XXX',F8.6)
0053      GOTO 333
0054      C
0055      LOOP FOR LARGE EXPONENTS
0056      C
0057      300 DO 322 K=1,C

```

PROBS1

```
0058      CSUMS=10.**(SUMLOG(NN)-SUMLOG(NN-K)-SUMLOG(K)
0059      1      +K*DLOG10(DBLE(P))+(NN-K)*DLOG10(DBLE(Q)))+CSUMS
0060      C      WRITE(6,501) CSUMS
0061      C 500      FORMAT(10X,F8.6)
0062      C 322 CONTINUE
0063      C
0064      C 333 BXLEC = CSUMS
0065      RETURN
0066      END
```

```

0001      C
0002      C
0003      C
0004      C
0005      SUBROUTINE PROBS2(NN,P,C,BXLEC)
0006      C
0007      THIS SUBROUTINE COMPUTES CUMULATIVE POISON
0008      C      PROBABILITIES
0009      C
0010      INTEGER C
0011      PP=P*NN
0012      TERM=1.0
0013      SUM=TERM
0014      C
0015      IF(C.EQ.0) GOTO 110
0016      DO 100 I=1,C
0017          TERM=TERM*PP/I
0018          SUM=SUM+TERM
0019      100 CONTINUE
0020      C
0021      110 BXLEC=SUM/EXP(PP)
0022      C
0023      RETURN
0024      END

```

```

0001      C
0002      C
0003      C
0004      C
0005      SUBROUTINE ASN(MC, NS, K, N11, KOPT, C1MIN, C2MIN, N1MIN, N2MIN, ASNMIN)
0006      C
0007      C      THIS SUBROUTINE COMPUTES ASN(PO) VALUES AND ASNMAX
0008      C      VALUES.
0009      C
0010      DOUBLE PRECISION SUMLOG
0011      INTEGER C1MIN, C2MIN
0012      COMMON/BLK1/N2S, N2L
0013      COMMON/BLK3/N1
0014      COMMON/BLK4/ALPHA, BETA
0015      COMMON/BLK5/PO, P1
0016      COMMON/BLK6/I1, I2
0017      COMMON/BLK7/SUMLOG(2500)
0018      COMMON/BLK8/N
0019      COMMON/BLK11/ASN, ASNMAX
0020      C
0021      C      INITIALIZATION
0022      C      COMPUTE P* (MAXIMUM PROB. FOR ASNMAX)
0023      C
0024      J=I1+1
0025      XXX=0.0
0026      IF(I1.GT.0) XXX=SUMLOG(I1)
0027      AKONST=10.**(SUMLOG(I2)+SUMLOG(N11-I2-1)-XXX-SUMLOG(N11-I1-1))
0028      TEMP=1.0/FLOAT(I2-I1)
0029      AKONST=AKONST**TEMP
0030      PSTAR=AKONST/(1.0+AKONST)
0031      IF(K.EQ.1) CALL PROBS1(N11, PSTAR, I2, BXLEC)
0032      IF(K.EQ.2) CALL PROBS2(N11, PSTAR, I2, BXLEC)
0033      TEMP=BXLEC
0034      IF(K.EQ.1) CALL PROBS1(N11, PSTAR, I1, BXLEC)
0035      IF(K.EQ.2) CALL PROBS2(N11, PSTAR, I1, BXLEC)
0036      TEMP1=TEMP-BXLEC
0037      ASNMAX=FLOAT(N11)+N2S*TEMP1
0038      C
0039      IF(K.EQ.1) CALL PROBS1(N11, PO, I2, BXLEC)
0040      IF(K.EQ.2) CALL PROBS2(N11, PO, I2, BXLEC)
0041      TEMP=BXLEC
0042      IF(K.EQ.1) CALL PROBS1(N11, PO, I1, BXLEC)
0043      IF(K.EQ.2) CALL PROBS2(N11, PO, I1, BXLEC)
0044      TEMP2=TEMP-BXLEC
0045      ASN=FLOAT(N11)+N2S*TEMP2
0046      IF(ASNMAX.GT.NS.OR.ASN.GT.ASNMIN) GOTO 100
0047      ASNMIN=ASN
0048      N1MIN=N11
0049      N2MIN=N2S
0050      C1MIN=I1
0051      C2MIN=I2
0052      C
0053      100 IF(KOPT.NE.2) GOTO 500
0054      WRITE(5,110) N11, N2S, TEMP1, ASNMAX, TEMP2, ASN
0055      110 FORMAT(/,10X, 2(I3,3X), 2(F6.4,3X, F8.4,3X))
0056      C
0057      C

```



END

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